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CALCULATION OF TURBULENT BOUNDARY LAYER FOR A SLOT JET IMPINGEMENT ON A FLAT SURFACE

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ABSTRACT

The objective of this study is to characterize flow parameters for two-dimensional turbulent jets impinging on a flat surface. An integral form of the momentum equation has been used to obtain a hydrodynamic solution. The boundary layer was divided into three regions, stagnation zone, developing zone and fully developed zone for free-surface and free shear, and into two regions, stagnation and wall jet zone for submerged jet configurations. A nonlinear ordinary differential equation has been obtained for frictional velocity at each zone using a logarithmic velocity profile with Coles's law of the wake and solved numerically to predict wall shear stress as well as boundary layer and momentum thicknesses. The proposed method is more straightforward and computationally less expensive in calculating the main flow parameters as compared to turbulent flow models such as RANS and LES. Predicted wall shear stresses for a submerged jet were compared to experimental data for different cases and showed agreement with experimental data.

INTRODUCTION

Impinging jets have been widely used in industry to enhance the process of heat and mass transfer. Thermal management is vital for electronic equipment and a challenging area for aerospace engineering and many other applications. Gas or liquid impinging jets are used to control the operating temperature of electronic circuits and their components. In aerospace engineering and turbine design, heat transfer and hydrodynamic calculations of jet impinged surfaces is of great importance. Controlling the mass transfer in jet deposition processes and erosion study of slurry jets are other examples of engineering problems which require prediction of the fluid behavior in jet impingement configurations. In addition to the direct use of a hydrodynamic solution of the impinged jets, heat transfer calculations are possible if velocity of the fluid near the wall is known. Slot jets are two-dimensional problems that are not only simplified models of real world applications but also serve as a starting point for modeling three-dimensional and other complex geometries.

Different configurations of impinging jets have been studied in the literature. They may be classified into three main categories: free-surface, submerged and confined. When the jetting fluid and surroundings are immiscible, a free-surface jet forms. A common example is liquid issued from a nozzle into a gas environment. In the submerged case, free shear flow is initiated at the exit of the nozzle, deflected as it approaches the wall and spreads out over the wall while entraining the surrounding fluid and being diffused into the surrounding fluid. Confined jet arrangement is similar to a submerged jet, except that a confining boundary parallel to the impinging surface limits the entrainment process of the jet and some kind of channel flow forms while the jet is spreading out. There are many studies in the literature on free-surface, submerged and confined impinging jets but most of them are for circular jets in axi-symmetric geometries. The studies may be classified as experimental, numerical, theoretical or combination of these.

Donaldson et al. (1971) measured the heat transfer characteristics of a circular impinging jet and introduced a correction term to use the laminar heat transfer coefficient for turbulent flows. Elison and Webb (1994) studied local heat transfer experimentally for a liquid jet impinging a flat surface with uniform heat flux. Womac et al. (1993) conducted experimental investigations of liquid jet impingement cooling of square heat sources in free-surface and submerged configurations as a simplified model of circuit chip heat transfer. Maurel and Solliec (2001) measured jet centerline mean velocity and velocity fluctuations in plane turbulent jets impinging to a flat surface and variety of stand-off distances

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using Laser Doppler Velocimetry (LDV) and Particle Image Velocimetry (PIV) techniques. Narayanan et al. (2004) investigated the flow field, surface pressure and heat transfer of a submerged turbulent slot impinging jet experimentally. They mainly focused on the impinging velocity field and heat transfer.

Looney and Walsh (1984) studied laminar free jet, turbulent free jet and turbulent impinging jet numerically. They solved Reynolds averaged Navier-Stokes equation using finite volume method and closure expressions for turbulent stresses. Agreement between predictions and experimental data in the literature has been found for most of the cases with some limitations on the turbulence models. Tong (2003) studied the liquid jet impinging onto a substrate numerically, and the effect of key parameters on the hydrodynamics and heat transfer of an impinging liquid jet were examined. The numerical results showed a good agreement with experimental data obtained from literature.

Phares et al. (2000) determined the wall shear stress theoretically under impinging jets of axi-symmetric and two-dimensional configurations. They assumed laminar boundary layer up to the transition point which is between the stagnation point and wall jet region irrespective of level of turbulence in the free stream and an empirical expression from literature has been used to estimate the wall mean shear stress for the turbulent region. Another theoretical study has been conducted by Chen et al. (2005) to present an expression for hydrodynamic and thermal boundary layer thicknesses and heat transfer characteristics of a free-surface liquid slot jet impingement. Although the results showed a good agreement with experimental data, this theory was obtained based on laminar boundary layer and is not valid for turbulent jets with high Reynolds numbers.

In this paper, an integral form of the Navier-Stokes equation has been used to characterize flow parameters of two-dimensional turbulent slot jets impinging on a flat surface in both free surface and submerged configurations. The flow field for each geometry is divided into simplified physical models to find the external velocity (which is the velocity outside boundary layer). Logarithmic law-of-the wall is used for the internal velocity profile to calculate integral parameters in the boundary layer. The result is a single ordinary nonlinear differential equation that can be solved numerically for frictional velocity. Other hydrodynamic parameters such as boundary layer, displacement and momentum thicknesses can be calculated from algebraic equations in term of frictional velocity.

NOMENCLATURE

B	Near-universal constant for turbulent flow
C_f	Friction factor
H	Standoff distance in submerged geometry
U_e	Free stream velocity
u_m	Maximum velocity at jet centerline before impingement; velocity at $y = \delta_m$
$\bar{u}$	Lateral velocity component
u^+	Inner variable form of lateral velocity
$\overline{\acute{u}\acute{v}}$	Reynolds stress
V	Initial jet velocity
$\bar{v}$	Normal velocity component
W	Slot nozzle width
x	Coordinate parallel to the impingement plane
y	Coordinate normal to the impingement plane
y^+	Inner variable form of y
δ	Boundary layer thickness
δ^*	Displacement thickness
θ	Momentum thickness
κ	Near-universal constant for turbulent flow
ν	Dynamic viscosity
Π	Wake parameter
τ_m	Shear stress at boundary
τ_W	Wall shear stress

THEORETICAL INVESTIGATION

Free-surface impinging jet

The physical model of the flow field for free-surface and submerged slot jets impinging a flat surface are shown in Fig. 1. The flow field along the target plane is divided into three regions for the free-surface configuration: stagnation zone, developing zone and fully developed zone. There are some models in the literature which can be combined to yield the solution in the specified regions. In the stagnation zone, pressure is decreasing from the maximum value at the origin which is equal to the stagnation pressure, while velocity is increasing along the x-axis. The two-dimensional incompressible boundary layer equations are reduced to (White, 2006)

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \qquad (1)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} \approx U_e\frac{dU_e}{dx} + \frac{\mu}{\rho}\frac{\partial \bar{u}}{\partial y} - \overline{\acute{u}\acute{v}} \qquad (2)$$

In which the pressure gradient term is replaced by velocity gradient using Bernoulli's relation. The integral momentum relation for turbulent flow was derived by Karman (1921) similar to the laminar flow relation. So, integrating Eq. (2) from the wall up to the boundary thickness, δ, in the vertical direction yields

$$\frac{d\theta}{dx} + \frac{(2\theta + \delta^*)}{U_e}\frac{dU_e}{dx} = \left(\frac{u^*}{U_e}\right)^2 \qquad (3)$$

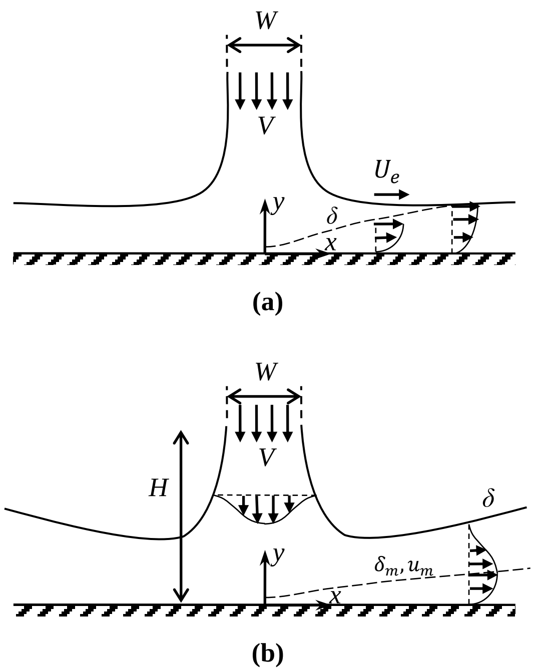

Figure 1. Physical representation of a) free-surface and b) submerged impinging jet

where

$$\theta = \int_0^{\delta} \frac{\bar{u}}{U_e}\left(1 - \frac{\bar{u}}{U_e}\right) dy \tag{4}$$

$$\delta^* = \int_0^{\delta} \left(1 - \frac{\bar{u}}{U_e}\right) dy \tag{5}$$

The approach in this paper will follow the inner variable theory (White, 2006) for unseparated turbulent flows. In order to solve Eq. (3), the velocity profile in the boundary layer needs to be known. The velocity profile selected to be the logarithmic law-of-the-wall and a polynomial form of the wake law which is originally Cole's law in the form of a sine function (Das, 1983)

$$u^+ \approx \frac{1}{\kappa}\ln(y^+) + B + \frac{2\Pi}{\kappa} f\left(\frac{y^+}{\delta^+}\right) \tag{6}$$

where

$$u^+ = \frac{\bar{u}}{u^*} \quad , \quad y^+ = \frac{y u^*}{\nu}, \delta^+ = \frac{\delta u^*}{\nu} \tag{7}$$

$$f\left(\frac{y^+}{\delta^+}\right) = 3\left(\frac{y^+}{\delta^+}\right)^2 - 2\left(\frac{y^+}{\delta^+}\right)^3 \tag{8}$$

The logarithmic law-of-the-wall has been verified for fully developed turbulent wall jet (region 3 in both configurations) in the literature (Bradshaw and Gee 1962) and it is assumed that it can be used as an approximation for the developing region as well.

Integrating Eq. (6) across the boundary layer using Eqs. (4) and (5) yields

$$\frac{\delta^*}{\delta} \approx \frac{1 + \Pi}{\kappa U^+} \tag{9}$$

$$\frac{\theta}{\delta} \approx \frac{\delta^*}{\delta} - \frac{2 + 3.17\Pi + 1.49\Pi^2}{\kappa^2 {U^+}^2} \tag{10}$$

where

$$U^+ = U_e / u^* \tag{11}$$

Substituting these equations in Eq. (3) results in a single ordinary nonlinear differential equation for $u^*(x)$ which can be solved numerically.

$$\frac{e^{\kappa\frac{U_e(x)}{u^*(x)} - B\kappa - 2\Pi}}{k^2\, u^*(x)^2}\{\kappa\, \nu U_e{}'(x)[\kappa(1+\Pi)U_e(x) - (1.17\Pi + 1.49\Pi^2)u^*(x)]\, u^*(x) + \nu[\kappa(2 + 3.17\Pi + 1.49\Pi^2)U_e(x)\, u^*(x) - \kappa^2(1+\Pi)U_e(x)^2 - (2 + 3.17\Pi + 1.49\Pi^2)\, u^*(x)^2]\, {u^*}'(x)\} = u^*(x)^2 \tag{12}$$

The free surface flow velocity, U_e may be approximated from inviscid solution. So, in the stagnation region

$$U_e = V.(x/W) \tag{13}$$

In this region, because of the strong favorable pressure gradient, the wake term in the velocity profile is negligible ($\Pi \approx 0$). In the developing region, where the boundary layer has not reached the surface, the free-stream velocity is almost constant and equal to the initial velocity, *V*. The problem becomes similar to uniform flow over a flat plate.

In the third region of the flow, the velocity profile is fully developed, viscous stresses become appreciable up to the surface. The boundary layer equation (Eq. 12) may be revised because the free surface velocity is unknown. The conservation of mass requires,

$$2\int_0^{\delta} \bar{u}\, dy = V \cdot W \tag{14}$$

so,

$$\frac{U_e}{u^*} = \frac{V\, W}{2\nu\, e^{\frac{\kappa U_e}{u^*} - B\kappa - 2\Pi}} \tag{15}$$

or

$$U_e = \frac{u^*}{\kappa}\left[\mathcal{W}\left(\frac{\kappa V W e^{B\kappa+\Pi-1}}{2\nu}\right)+\Pi+1\right] \quad (16)$$

where $\mathcal{W}(\dots)$ is Lambert W Function. So, substituting Eq. (16) in Eq. (12) gives a single nonlinear ordinary differential equation in $u^*(x)$.

Submerged impinging jet

A physical model of the submerged jet is shown in Fig. 1-b. The flow phenomena can be characterized by three flow regions: free jet region, stagnation region and wall jet region. The free jet exits from the slot with a width of W and issues into a still stream. Mixing layers form on the two sides and grow between the inviscid potential core at the same velocity as the nozzle exit velocity, V, and ambient fluid. The standoff distance, H, determines if the potential core reaches the target plate or not. According to an experimental study by Cadek (1968), in the stagnation region the inertia term outweighs the viscous terms specifically for $H/W < 4$; the stagnation region almost falls within the free jet potential core. The jet centerline velocity, u_m, may be approximated from the inviscid flow solution, and the rest of the calculations are similar to the free-surface configuration except the free-surface velocity, U_e, is replaced by u_m. In the wall jet region, the pressure remains almost constant, and for $H/W > 4$, the jet centerline velocity decays by the following correlation,

$$\frac{u_m}{V} = \frac{2.36}{\sqrt{(x+H)/W}} \quad (17)$$

The momentum equation for the inner layer reduces to

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} \approx \frac{1}{\rho}\frac{\partial \tau}{\partial y} \quad (18)$$

Integrating from $y = 0$ to δ_m and combining with the continuity equation yield

$$\int_0^{\delta_m}\frac{\partial u^2}{\partial x}dy - u_m\int_0^{\delta_m}\frac{\partial u}{\partial x}dy = \frac{\tau_m}{\rho} - \frac{\tau_W}{\rho} \quad (19)$$

τ_m is the shear stress at $y = \delta_m$ and equal to $-\tau_W$ from measurements by Bradshaw and Gee (1962). The logarithmic velocity profile has been used and Leibnitz' rule has been applied to obtain the final ordinary differential equation for $u^*(x)$. Eq. (20)

This equation is similar to Eq. (12) except that the pressure term is not taken into account and friction is introduced both at the lower and upper boundaries.

$$\frac{e^{\kappa\frac{u_m(x)}{u^*(x)}-B\kappa-2\Pi}}{k^2\,u^*(x)^2}\{\kappa\,\nu u_m{}'(x)[\kappa\Pi u_m(x) - (1.17\Pi+1.49\Pi^2)u^*(x)]\,u^*(x) + \nu[\kappa(2+3.17\Pi+1.49\Pi^2)u_m(x)\,u^*(x) - \kappa^2(1+\Pi)u_m(x)^2 - (2+3.17\Pi + 1.49\Pi^2)\,u^*(x)^2]\,u^{*\prime}(x)\} = 2\,u^*(x)^2 \quad (20)$$

After calculating the frictional velocity for each case using the developed differential equation, other hydrodynamic parameters such as wall shear stress and boundary layer, displacement and momentum thicknesses can be calculated from the following relations and Eqs. (9,10) .

$$\tau_W = \rho u^{*2} \quad (21)$$

$$\delta = \frac{\nu e^{\frac{\kappa U_e}{u^*}-B\kappa-2\Pi}}{u^*} \quad (22)$$

COMPARISON WITH EXPERIMENTAL DATA

In order to evaluate the performance of the developed equations, the wall shear stress predictions have been compared to experimental data from literature for a submerged impinging slot jet since experimental data for this configuration are more frequent than for the free-surface jet configuration.

Tu and Wood (1996) measured wall pressure and surface shear stress for a two-dimensional turbulent impinging jet using Preston and Stanton probes at different ratios of impingement height (H) to nozzle width (W). The calibration process of these probes requires special flow condition and they are usually calibrated in a well-developed turbulent boundary layer that logarithmic law applies outside the viscous sublayer. Various sizes of probes were used to measure the shear stress in the viscous sublayer and the smaller probes measured higher wall shear stress especially in the developing zone. It was shown that measurements with Stanton probe even thicker than the viscous sublayer at stagnation are not affected by the probe size.

A comparison of results for three cases, $H/W = 2, 4, 12$ is provided in Fig. 2 in which the friction factor (C_f) is defined as

$$C_f = \frac{\tau_w}{\rho V^2/2} = 2\left(\frac{u^*}{V}\right)^2 \quad (23)$$

In their experiment, air is selected as the working fluid with different Reynolds numbers. It is assumed that turbulent flows are in an equilibrium state and the wake term, Π, is constant. As depicted in the figure, the present study predictions follow the trend of experimental values especially for H/W = *2* and H/W = 4 and in the stagnation zone but generally under-predicts the friction coefficient in the wall jet zone. For these two cases, the potential core reaches the target surface, so the inviscid flow solution in the stagnation region is more acceptable as compared to the case where H/W = 12. The step increase in the predicted values for H/W = 2 is due to the assumption of constant external velocity before the first point that self-similar correlation is applicable. The discontinuity in the prediction values is due to the approximation of external velocity with inviscid flow solution and dividing the flow field into two distinct regions, stagnation and wall jet. Experimental values of the outer flow pressure or velocity may be used in Eq. (20) to provide improved and continuous profile of the boundary layer parameters.

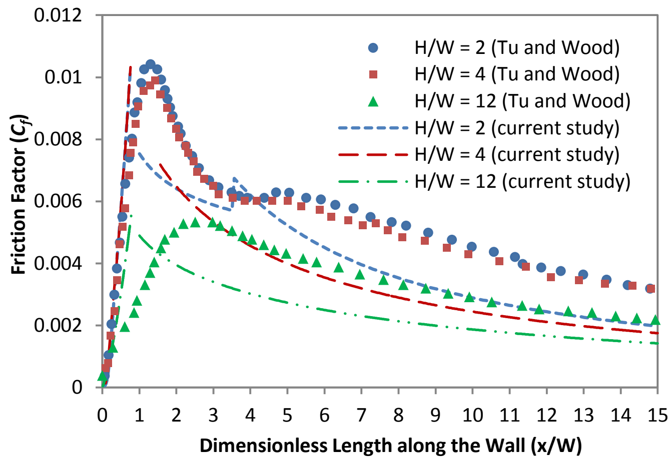

Figure 2. Comparison of present study predictions to experimental data (Tu and Wood 1996) for Re = 11,000 and H/W = 2, 4 and 12

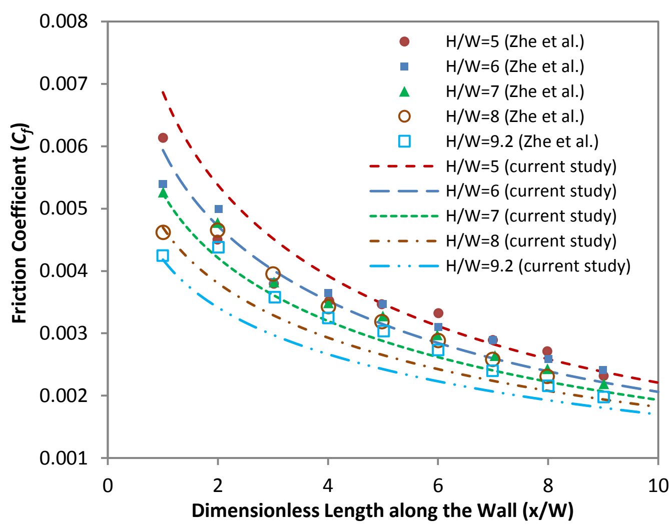

Figure 3. Comparison of present study predictions to experimental data (Zhe and Modi 2001) for Re = 20,000 and H/W = 5, 6, 7, 8, 9.2

In another experimental study in the literature, Zhe and Modi (2001) measured mean velocity and root mean square velocity using hot wire anemometry. They calibrated the hot-wire with manometric measurement of the free stream velocity in a Blasius flow to resolve the problem of interference with the wall and low velocity calibration. The uncertainty in the calculated shear stress is reported to be 14%. Figure 3 shows comparison of experimental data from Zhe and Modi (2000) with corresponding predictions from Eq. (20).

The predicted values are close to the experimental data and except the case H/W = 5, slightly under-predicts the shear along the surface for all other case.

CONCLUSIONS

An integral method has been implemented to calculate hydrodynamic parameters of a two-dimensional turbulent impinging jet on to a flat surface. A physical models for free-surface and submerged configurations have been obtained using simplifying assumptions. A nonlinear ordinary differential equation has been extracted for each region of the flow using logarithmic law-of-the-wall with Cole's wake function. The outer flow velocity has been approximated using an inviscid flow solution for stagnation and developing zones of the free-surface geometry and stagnation zone for the submerged geometry. The jet maximum centerline velocity in the wall jet region of the submerged impinging jet was assumed to follow the correlation for free shear jet flow. The external flow field has been approximated using inviscid flow relations or correlation for simplified geometries. Numerical solution of the differential equation is straightforward and computationally less expensive than methods used in other turbulent flow models such as RANS and LES. The wall shear stress prediction result for submerged configuration has been compared to experimental data from literature and showed fair agreement. At low distance to with ratios (H/W = 2, 4) in the stagnation region, the predicted values from this study are close to the experimental data and its deviation become considerable at high distance ratio (H/W = 12). But in the other hand, wall jet assumption and logarithmic velocity profile is more valid for high distance ratio configurations and the approximation of the external flow velocity from the self-similar correlation is more close to the real external velocity.

The predictions may be improved by implementing experimental values of the outer flow pressure or velocity. So, this method seems to be applicable to turbulent impinging jets and calculation of hydrodynamic and heat transfer parameters for other geometries is proposed as future work.

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